HALL ORCHARD

CE Primary School

Calculation Guidance

January 2020
Review January 2023

Progression from Mental Methods to Written Methods for Addition

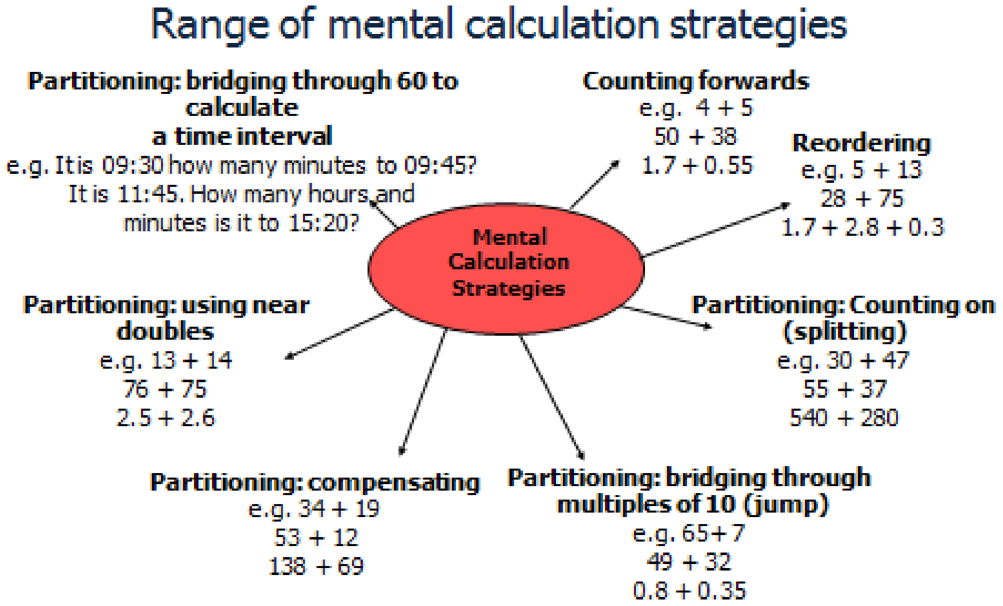
To add successfully, children need to be able to:

- count and have an understanding of quantity including greater than and less than symbols
- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways

Mental Skills

Recognise the size and position of numbers
Count on in ones and tens
Know number bonds to 10 and 20
Add multiples of 10 to any number
Partition and recombine numbers
Bridge through 10

It is important that children's mental methods of calculation are practiced and secured alongside their learning of written methods. The children should look at the numbers in the calculation and decide which is the most efficient method to use; mental, jotting or written.



Range of mental calculation strategies

Mental Calculation Strategies

Partitioning: bridging through 60 to calculate a time interval
e.g. It is 09:30 how many minutes to 09:45?
It is 11:45. How many hours and minutes is it to 15:20?

Counting forwards
e.g. 4 + 5
50 + 38
1.7 + 0.55

Reordering
e.g. 5 + 13
28 + 75
1.7 + 2.8 + 0.3

Partitioning: using near doubles
e.g. 13 + 14
76 + 75
2.5 + 2.6

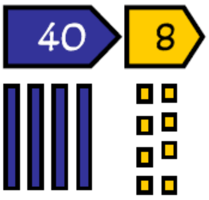
Partitioning: Counting on (splitting)
e.g. 30 + 47
55 + 37
540 + 280

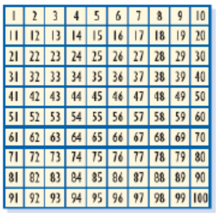
Partitioning: compensating
e.g. 34 + 19
53 + 12
138 + 69

Partitioning: bridging through multiples of 10 (jump)
e.g. 65+7
49 + 32
0.8 + 0.35

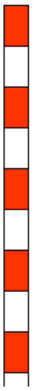
Concrete resources and visual representations

Counting apparatus
Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string
Numicon including software
NCETM 'Small–step teaching guidance'





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



EYFS

5 plus 3 = 8
3 more than 5 =8

Numicon shapes are introduced straight away and can be used to:
Identify 1 more/less
Combine pieces to add
Find number bonds
Add without counting
Begin to recognise combinations of odd and even
Children can record by mark making, printing or drawing around Numicon Shapes.

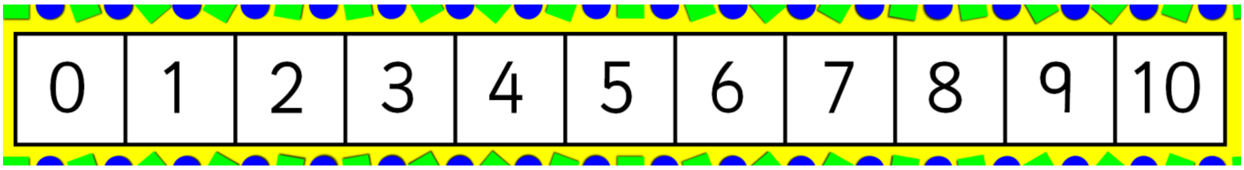
Children begin to combine groups of objects using concrete apparatus

Construct number sentences verbally or using cards to go with practical activities.

Children make a record in pictures, words or symbols of addition activities already carried out.

Solve simple problems using fingers

Number tracks can be introduces to count up on and to find one more:



What is 1 more than 4? 1 more than 3?

Children will need opportunities to look at and talk about different concrete resources and pictorial representations of quantities to develop an understanding of number

Vocabulary

Games and songs can be useful way to begin using vocabulary involved in addition e.g. Alice the camel

Add, more, and, make, sum, total, altogether, score, double, one more, two more, ten more

Year 1

+ = signs and missing numbers

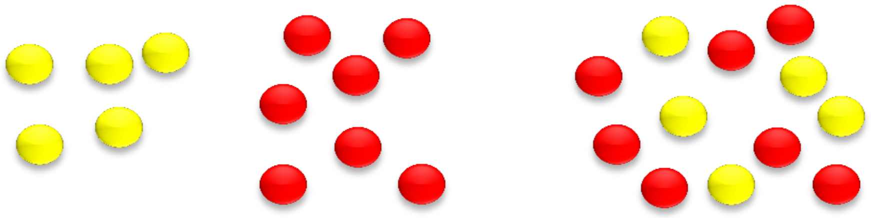
Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1 + 1   2 + 3 = 4 + 1

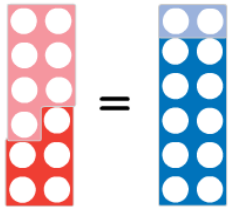
Missing numbers need to be placed in all possible places.

3 + 4 = □   □ = 3 + 4   3 + □ = 7   7 = □ + 4

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)



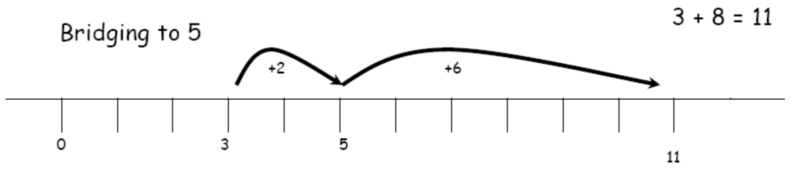
5 + 7 = 12

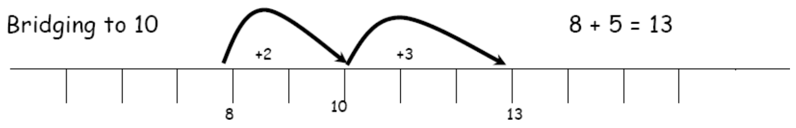


Understanding of counting on with a number track 0 - 20

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Understanding of counting on with a number line (supported by concrete resources and pictorial representations)





Vocabulary

Addition, add, forwards, put together, more than, total, altogether, equals = same as, most, pattern, odd, even, digit, counting on, double, near double.

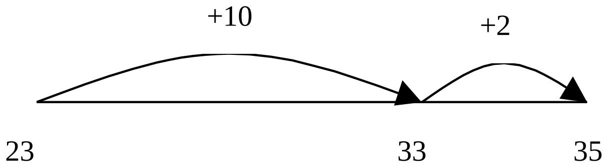
Year 2

Missing number problems

14 + 5 = 10 + □   32 + □ + □ = 100   35 = 1 + □ + 5

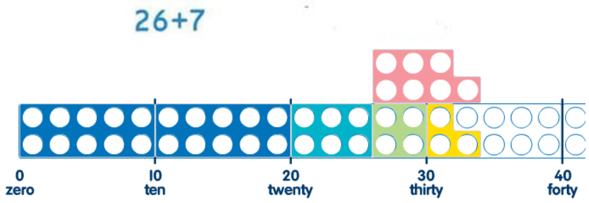
Counting on into tens and ones (units)

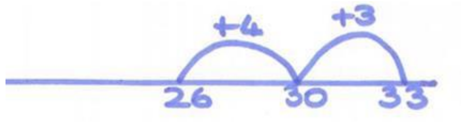
23 + 12 = 23 + 10 + 2



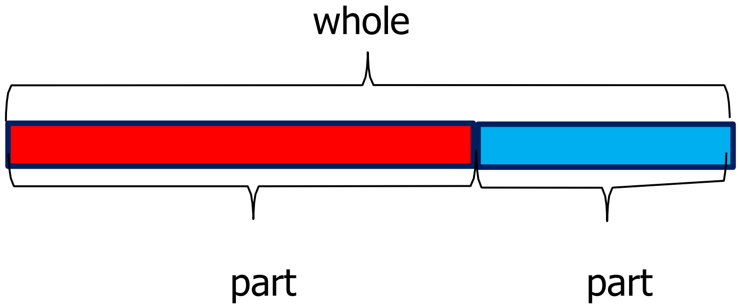
Partitioning and bridging through 10

The steps in addition often bridge through a multiple of 10





Part-whole bar model

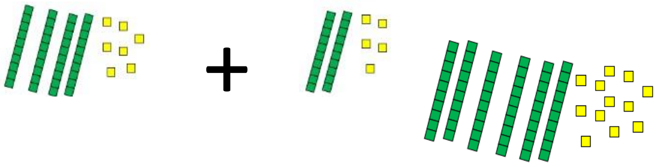


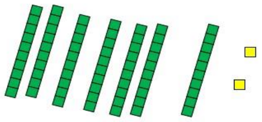
The bar model should used in the context of **measures and problem solving**

Towards a Written Method

Partitioning in different ways and recombing

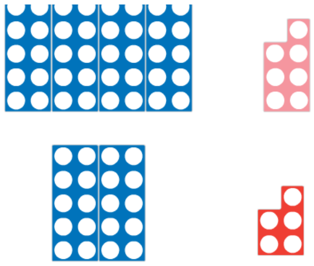
47 + 25 = 60 + 12   Leading to exchanging: 72





Expanded written method

  40 + 7
+ 20 + 5
  60 + 12 = 72



60 + 12

Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, tens, ones, partition, near multiple of 10, tens boundary, more than, one more, two more… ten more… one hundred more

Year 3

Missing number problems

140 + 150 = □ 320 + □ + 9 = 1000 521 + □ = 600

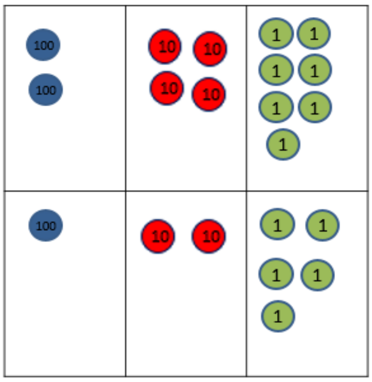
Partition into tens and ones

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20+ 5

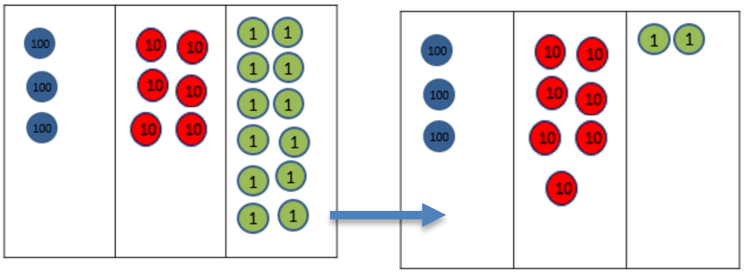
Towards a written method

Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



$$\begin{array}{l} 200 + 40 + 7 \\ \underline{100 + 20 + 5} \\ 300 + 60 + 12 = 372 \end{array}$$

Leading to children understanding the exchange between tens and ones



$$\begin{array}{r} 247 \\ \underline{+125} \\ 12 \\ 60 \\ \underline{300} \\ 372 \end{array} \longrightarrow \begin{array}{r} 247 \\ \underline{+\ 125} \\ \underline{\ \ 372} \\ {\scriptstyle 1} \end{array}$$

The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

Year 4

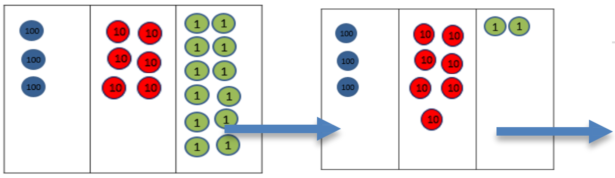
Missing number problems

6.5 + 2.7 = □   4087 + □ = 5000   7.2 + □ = 9

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

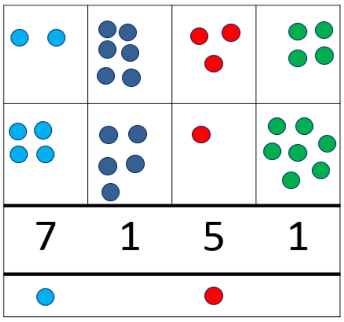


```
  247
 +125
   12
   60
  300
  372
```

```
   247
 + 125
   372
    1
```

Compact written method

Extend to numbers with at least four digits.



```
  2634
 +4517
  7151
  1  1
```

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits)

```
    72.8
  + 54.6
   127.4
    1
```

Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones / tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? equals sign, is the same as

Year 5

Missing number problems

12.43 + 9.81 = □   24087 + □ = 35000   7.26 + □ = 9.52

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency.

Written methods (progressing to more than 4-digits)

Children calculate using the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

$$\begin{array}{rr} & \mathbf{172.83} \\ + & \underline{\mathbf{54.68}} \\ & \underline{\mathbf{227.51}} \\ & {\scriptstyle \mathbf{1\ \ 1\ \ 1}\ \ \ \ \ } \end{array}$$

Concrete resources can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Vocabulary

Tens of thousands boundary plus all previous vocabulary

Year 6

Missing number problems

12.43 + 19.8 = □   23507 + □ = 35429   7.26 + □ = 29.5

Mental methods should continue to develop, supported by a range of concrete resources and pictorial representations, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency.

Written methods

As year 5, progressing to larger numbers; aiming for both conceptual understanding and procedural fluency with a secure columnar method.

$$\begin{array}{rr} & \mathbf{172.83} \\ + & \underline{\mathbf{54.68}} \\ & \underline{\mathbf{227.51}} \\ & {\scriptstyle \mathbf{1\ \ 1\ 1}\ \ \ } \end{array}$$

Continue calculating with decimals, including those with different numbers of decimal places.

Vocabulary

Consolidation of previous years

Progression from Mental Methods to Written Methods for Subtraction

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20
- subtract multiples of 10 (such as 160 - 70) using the related subtraction fact, 16 - 7, and their knowledge of place value
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Mental Skills

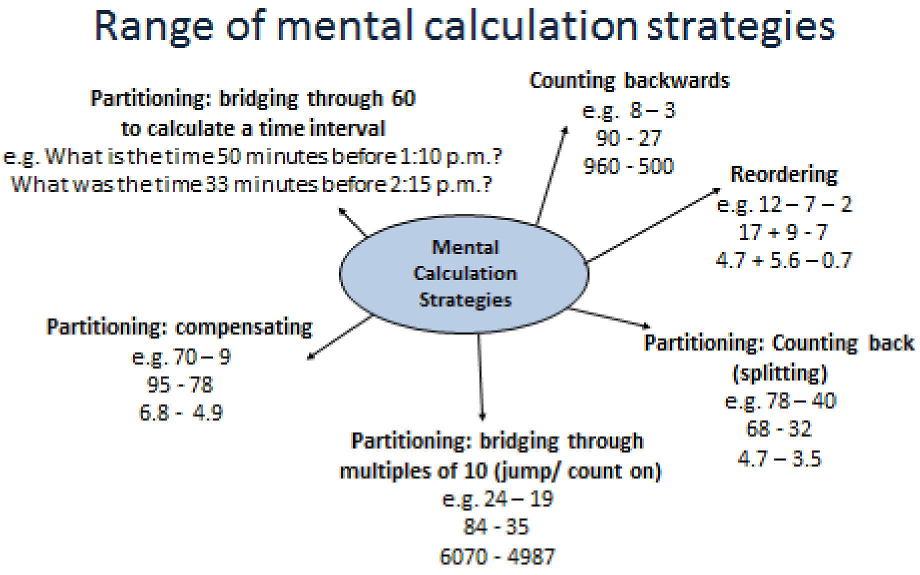
Recognise the size and position of numbers Count back in ones and tens

Know number facts for all numbers to 20. Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Bridge through 10

It is important that children's mental methods of calculation are practised and secured alongside their learning of written methods. The children should decide which is the most efficient method to use e.g. mental, jotting or written.



Range of mental calculation strategies

Mental Calculation Strategies

Partitioning: bridging through 60 to calculate a time interval
e.g. What is the time 50 minutes before 1:10 p.m.?
What was the time 33 minutes before 2:15 p.m.?

Counting backwards
e.g. 8 – 3
90 - 27
960 - 500

Reordering
e.g. 12 – 7 – 2
17 + 9 - 7
4.7 + 5.6 – 0.7

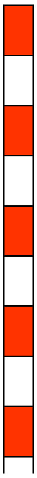
Partitioning: compensating
e.g. 70 – 9
95 - 78
6.8 - 4.9

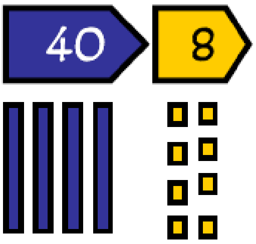
Partitioning: Counting back (splitting)
e.g. 78 – 40
68 - 32
4.7 – 3.5

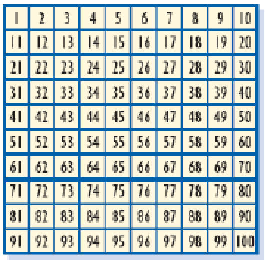
Partitioning: bridging through multiples of 10 (jump/ count on)
e.g. 24 – 19
84 - 35
6070 - 4987

Concrete resources and visual representations

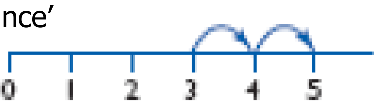
Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered lines
Hundred square
Empty number lines.
Counting stick
Bead strings
Numicon including software
NCETM 'Small–step teaching guidance'







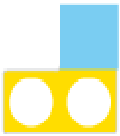
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



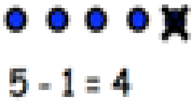
EYFS

Children begin with concrete resources and record mostly with pictorial representations (not necessarily using numerals and symbols)

Remove some objects and count



Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.



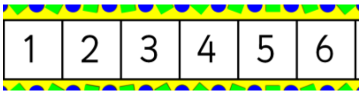
Construct number sentences verbally or using cards to go with practical activities.

6 fingers up. How many are down?
4 fingers are down. How many are up?

Children are encouraged to read number sentences aloud in different ways, "five subtract one leaves four," "four is equal to five subtract one."

Children make a record in pictures, words or symbols of subtraction activities already carried out.

Number tracks and Numicon Shapes can be introduced to count back to find one less.



What is 1 less than 9? 1 less than 20?

Children will need opportunities to look at and talk about different concrete resources and pictorial representations of quantities to develop an understanding of number.

Vocabulary

Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g. Five little speckled frogs

Take (away), leave, how many are left/left over?, how many have gone?, one less, two less etc., how many fewer is . . . than . . . ?, difference between, is the same as

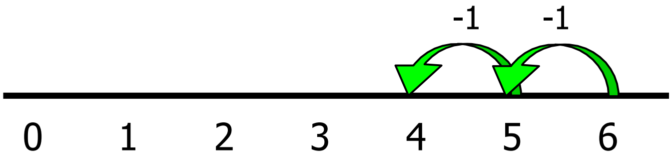
Year 1

– = signs and missing numbers

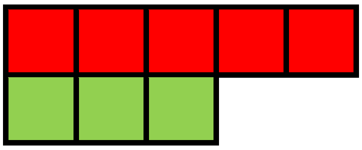
7 = ☐ – 9 20 - ☐ = 9 ☐ - ☐ = 11 16 – 0 = ☐

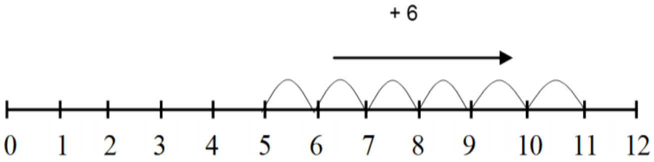
Children need to use concrete objects and pictorial representations. Support them to use number lines with every number shown to number lines with significant numbers shown. It is advisable to provide the children with the number lines rather than expect them to draw their own.

Understand subtraction as take-away:



Understand subtraction as finding the difference:





The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to a pictorial representation.

The use of other concrete resources is also valuable for modelling subtraction e.g. Numicon Shapes, bundles of straws, multi-link cubes, bead strings

Vocabulary

Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Year 2

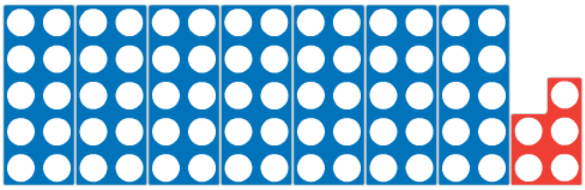
Missing number problems

52 – 8 = ☐ ☐ – 20 = 25 22 = ☐ – 21 6 + ☐ + 3 = 11

Take away and difference

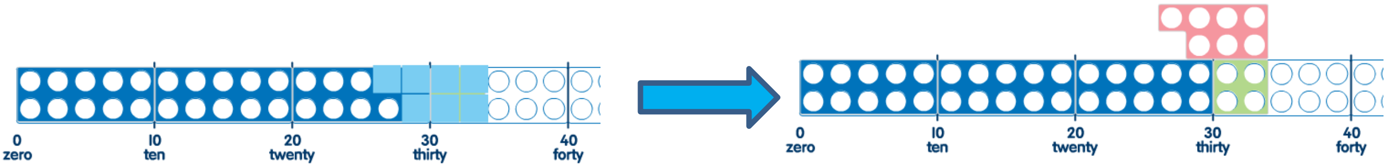
Children use concrete objects and pictorial representations. Continue to use number lines to model take-away and difference.

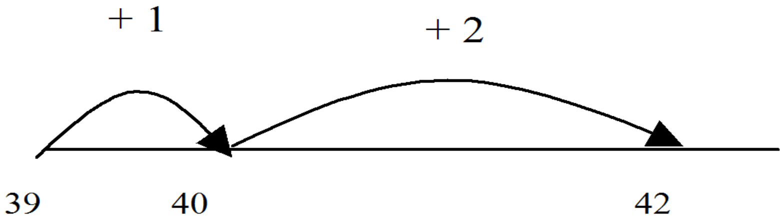
75 - 42



\- 42

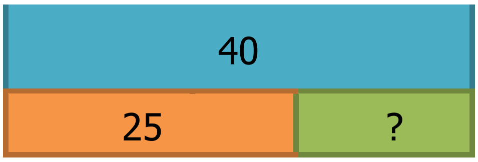
34 – 7





The comparison bar model should continue to be used, as well as images in the context of **measures and problem solving**.

Janie has 40 beads. She loses 25 of them. How many does she have left?



40 = 25 + ☐

40 = ☐ + 25

40 – 25 = ☐

40 - ☐ = 25

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus, tens, ones, partition, near multiple of 10, tens boundary, less than, one less, two less… ten less… one hundred less, more, one more, two more... ten more... one hundred more

Year 3

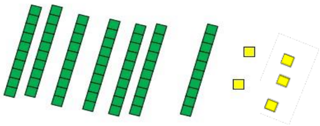
Missing number problems

☐ = 43 – 27　　145 – ☐ = 138　　364 – 153 = ☐

Towards written methods

Children should make choices about whether to use complementary addition (difference) or counting back (take away), depending on the numbers involved.

Recording subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus or Numicon e.g. children make the 75, they then remove 47. They then identify the answer. The children will record each step as they practically solve the calculation



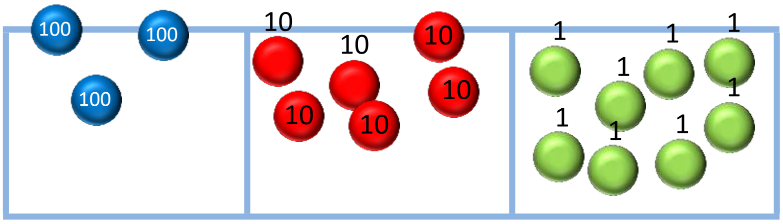
– 47

	70	5
-	40	7
	20	8

It is important that children experience 'exchanging' a ten into ones to support the understanding of decomposition before the more formal written methods are introduced.

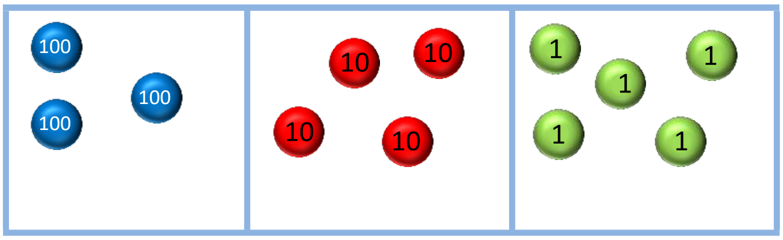
Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition initially, modelled with place value counters (Dienes could be used for those who need a less abstract representation).



	300	60	8
-	100	40	5
	200	20	3

Some children may begin to progress towards a more formal method that shows the exchange (**you should introduce this by using two digit numbers initially**). You will notice that the digits are crossed out and replaced with the exchanged values.



		30	15
	300	~~40~~	~~5~~
-	100	20	9
	200	10	6

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

Year 4

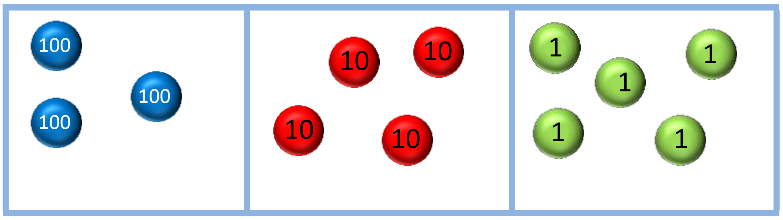
Missing number problems

☐ = 434 – 273 200 – 90 – 80 = ☐ 225 - ☐ = 150 ☐ - 2000 = 900

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

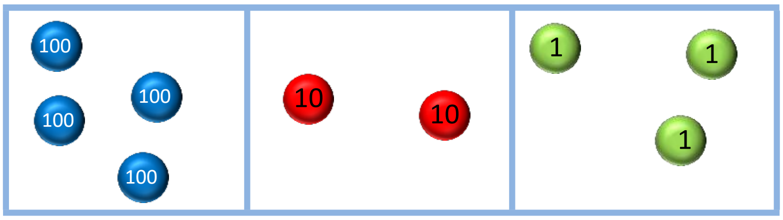
Written methods (progressing to 4-digits)

Some children may still need to consolidate their understanding of subtraction and may need to **repeat the Year 3 approach without any exchange** (decomposition). You will notice that the digits are crossed out and replaced with the exchanged values.



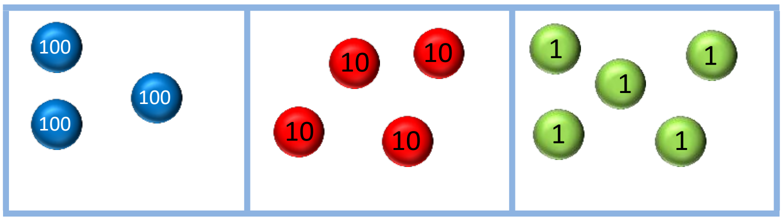
$$\begin{array}{rrr} 300 & \overset{30}{\cancel{40}} & \overset{15}{\cancel{5}} \\ -\ 100 & 20 & 9 \\ \hline 200 & 10 & 6 \end{array}$$

Continue to support their understanding of exchange through a double exchange worked through practically with expanded notation.



$$\begin{array}{rrr} \overset{300}{\cancel{400}} & \overset{110}{\cancel{20}} & \overset{13}{\cancel{3}} \\ -\ 100 & 80 & 9 \\ \hline 200 & 30 & 4 \end{array}$$

The formal compact method should be introduced as a more streamlined version of the expanded method, not a new method. Children can still use place value counters to support their understanding and as with early development stages you should drop back a stage (no decomposition) to secure the understanding before moving on.



$$\begin{array}{r} 3\overset{3}{\cancel{4}}\overset{15}{\cancel{5}} \\ -\ 129 \\ \hline 216 \end{array}$$

Extend to 4-digit number expanded column subtraction with decomposition, modelled with place value counters. Some children may progress further and be able to use the compact method.

Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Year 5

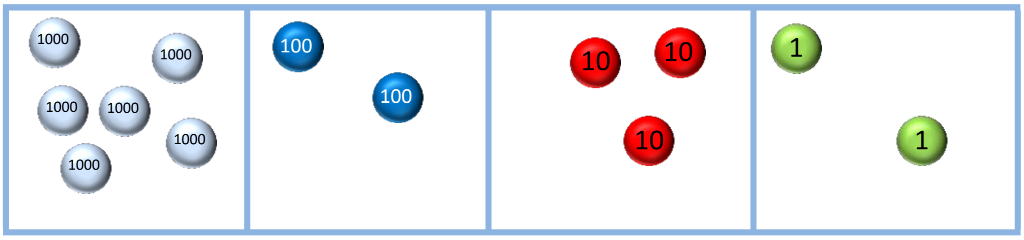
Missing number problems

1 000 000 - ☐ = 999 000 12 462 – 2 300 = ☐ 6.45 = 6 + 0.4 + ☐

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency.

Written methods (progressing to more than 4-digits)

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters. You will notice that the digits are crossed out and replaced with the exchanged values.



$$\begin{array}{r} 5\ 12\ 2\ 12 \\ \cancel{6}\cancel{2}\cancel{3}\cancel{2} \\ \underline{-\ 4814} \\ \underline{\quad 1418} \end{array}$$

Progress to calculating with decimals, including those with different numbers of decimal places by using place value counters until the concept is secure.

When children fully understand the use of the superscript they can progress to the more recognised form as modelled below. Extend with larger numbers.

$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}2{}^{2}\cancel{3}{}^{1}2 \\ \underline{-\ 4\ 8\ 1\ 4} \\ \underline{\quad 1\ 4\ 1\ 8} \end{array}$$

Vocabulary

tens of thousands boundary, also consolidation of previous years

Year 6

Missing number problems

10 000 000 = 9 000 100 + ☐   (7 – 2) x 3 = ☐   (☐ - 2) x 3 = 15

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency.

Written methods

As year 5, progressing to larger numbers; aiming for both conceptual understanding and procedural fluency with a secure columnar method.

$$\begin{array}{r} {}^{5}\cancel{6}\;{}^{1}2\;{}^{2}\cancel{3}\;{}^{1}2 \\ -\;4\;\;8\;\;1\;\;4 \\ \hline 1\;\;4\;\;1\;\;8 \\ \hline \end{array}$$

Continue calculating with decimals, including those with different numbers of decimal places.

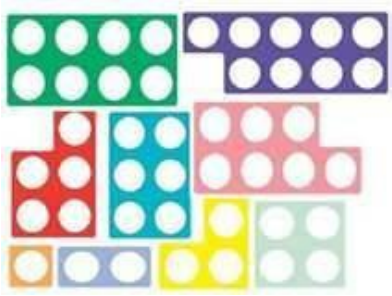
Vocabulary

Consolidate previous years

Progression from Mental Methods to Written Methods for Multiplication

To multiply successfully, children need to be able to:

- recall all multiplication facts to 12 × 12
- partition number into multiples of one hundred, ten and one
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value

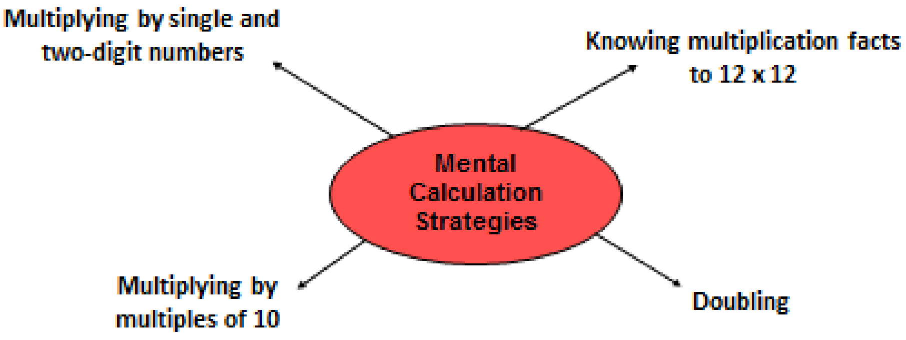


Mental Skills

Recognise the size and position of numbers
Count on in different steps 2s, 5s, 10s
Double numbers up to 10
Recognise multiplication as repeated addition
Quick recall of multiplication facts
Use known facts to derive associated division facts
Use known facts to generate other facts (e.g. double the 2 x table to find 4 x table)
Multiplying by 10, 100, 1000 and understanding the effect

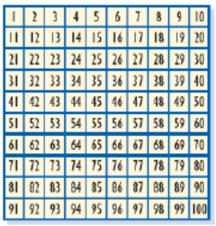
It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication. The children should decide the most efficient method to use, e.g. mental, jotting or written.

Range of mental calculation strategies



Concrete resources and visual representations



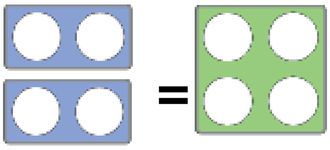


Counting apparatus
Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string
Numicon including software
100 square
Multiplication squares
Resources available from Times Tables Rock Stars and NCETM ‘Small–step teaching guidance’

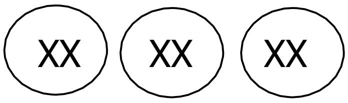
EYFS

Children need to have an understanding of addition; the concept of combining groups of objects and have grasped some basic addition skills.

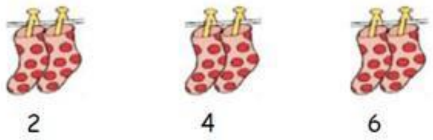
In the first instance the link between addition and multiplication can be introduced through **doubling**. Numicon is used to visualise the repeated addition of the same number. These can be drawn around or printed as a way of recording.



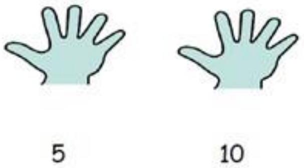
Children begin with mostly pictorial representations. How many groups of 2 are there?



The children will experience equal groups of objects and will count in 2s, 5s and 10s both aloud and with objects. They will work on practical problems solving activities involving equal sets or groups.



Children are given multiplication problems set in real life context. Children are encouraged to visualise the problem. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?



Progressing to **count in repeated groups of the same size** rather than counting in ones.

"How many wheels are there altogether?"

Vocabulary

How many sets of . . . ?, groups of, double

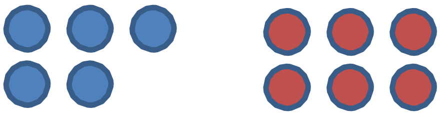
Year 1

Developing understanding

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

They should memorise and reason with numbers in 2, 5 and 10 times tables

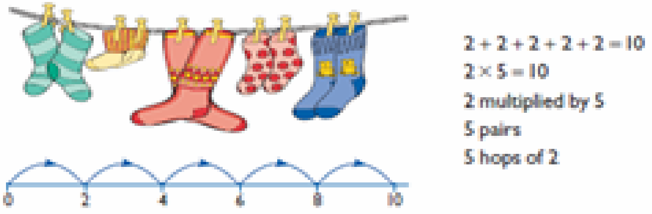
The children should also see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.



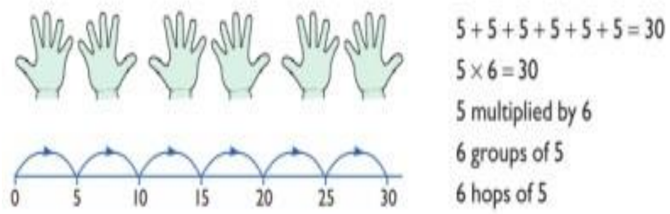
They should begin to understand multiplication as scaling in terms of double and half (e.g. that tower of cubes is double the height of the other tower).

Understanding multiplication as repeated addition

Children should understand that multiplication is related to doubling and combing groups of the same size (repeated addition).



2 + 2 + 2 + 2 + 2 = 10
2 × 5 = 10
2 multiplied by 5
5 pairs
5 hops of 2



5 + 5 + 5 + 5 + 5 + 5 = 30
5 × 6 = 30
5 multiplied by 6
6 groups of 5
6 hops of 5

When we model repeated addition we must ensure that we record the multiplication sentence correctly, i.e. the number in the group x the number of groups (read 'x' as multiplied by).

Problem solving

Problem solving with concrete objects including money and measures

"How much money do I have?"

Vocabulary

Ones, groups of, lots of, doubling, repeated addition, double

Year 2

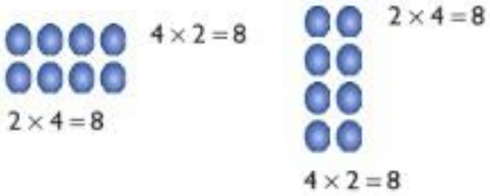
Developing understanding

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.
Number lines and Numicon Shapes should continue to be an important image to support thinking.
Children should practice times table facts accompanied by visual representations.
Use a clock face to support understanding of counting in 5s.
Use money to support counting in 2s, 5s, 10s, 20s, 50s.

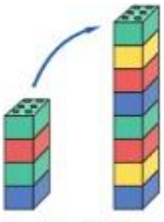
Children should begin to express multiplication as a number sentence using x.
They should use understanding of the inverse and practical resources to solve missing number problems, e.g.

7 x 2 = □ □ = 2 x 7 □ x 2 = 14 14 = 2 x □

They should use known facts to derive new facts e.g. double 15 = double 10 + double 5.

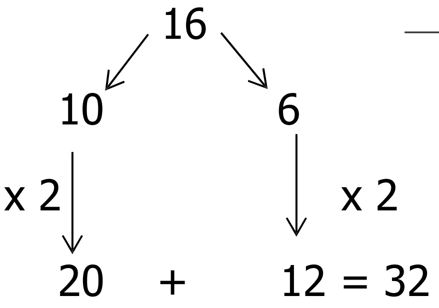


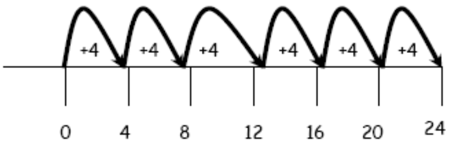
Children should be given plenty of opportunities to explore arrays. Include multiplications not in the 2, 5 or 10 times tables.
They should begin to develop understanding of multiplication as scaling (twice as tall, 3 times bigger).
By exploring Numicon Shapes, children should develop an understanding of the patterns within times tables, e.g. which tables contain only even numbers?

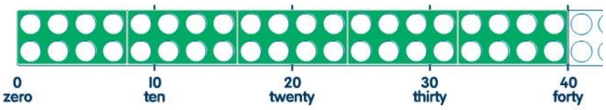

double 4 is 8
4 × 2 = 8

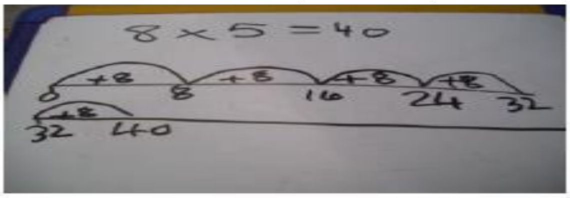
Towards written methods

Use number lines and jottings to develop an understanding of doubling two digit numbers and solving multiplication as repeated addition.









Vocabulary

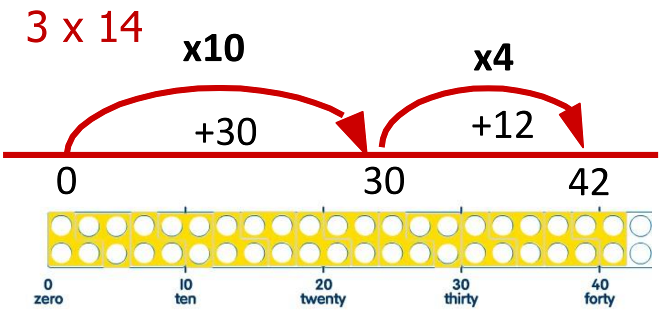
Multiple, multiplication array, multiplication tables/facts, columns, rows, times, multiply, multiplied by, multiple of

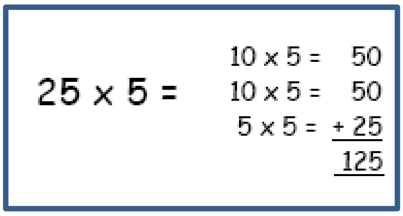
Year 3

Developing understanding

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

Practical resources and the number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged (see year 2). The children will continue to choose and use partitioning methods.




25 x 5 =

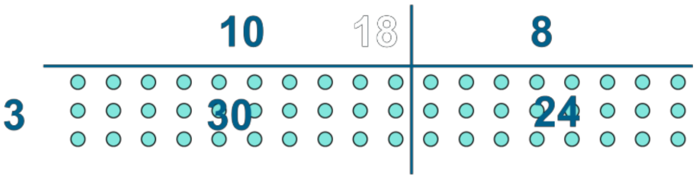
10 x 5 = 50
10 x 5 = 50
5 x 5 = + 25
125

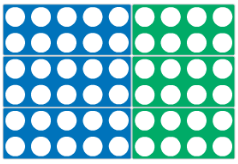
They will progress to jumping in larger groups which will support their partitioning for the grid methods.
The children should practise times table facts and continue to notice patterns.
They should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

7 x 4 = ☐ ☐ = 4 x 7 ☐ x 4 = 28 28 = 4 x ☐

Written methods (progressing to 2d x 1d)

Develop written methods using understanding of visual images and arrays.





When this understanding is in place progress to the grid method.

	10	8
3	30	24

Give the children plenty of opportunities to explore this and deepen their understanding by using Dienes apparatus and place value counters.

Vocabulary

Partition, grid method, inverse

Year 4

Developing understanding

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

They become fluent and confident to recall all tables up to 12 x 12.

They can multiply 3 small numbers together.

They should be encouraged to choose from a range of strategies:

- Partitioning using x 10, x 20 etc.
- Doubling to solve x 2, x 4, x 8
- Recall of times tables
- Use of commutativity of multiplication

They continue to use practical resources and the number line as an important image and informal jottings to support their thinking.

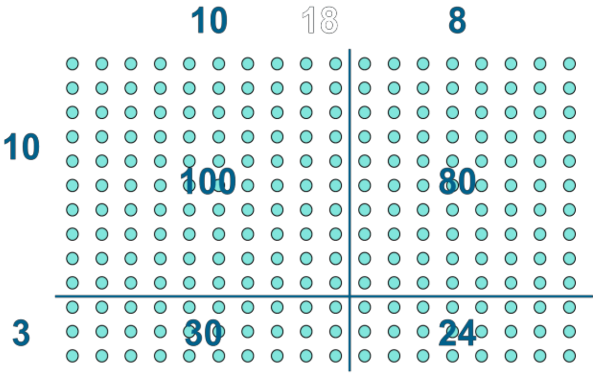
They should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

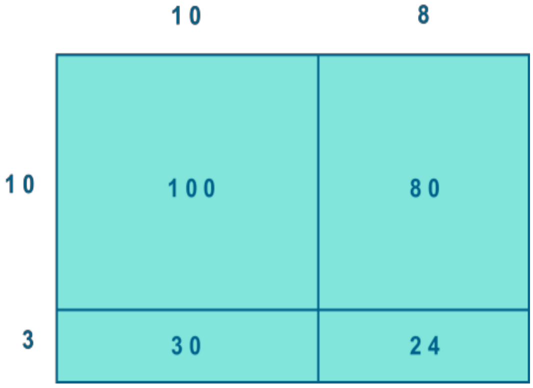
☐ 2 x 5 = 160   32 x ☐ = 160   ☐ ☐ x 8 = 160   ☐ ☐ x 4 = 160

They solve practical problems where they need to scale up using known number facts, e.g. how tall would a 25cm sunflower be if it grew 6 times taller?

Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply 2d x 2d, progressing to 3d x 2d. Ensure this is still linked back to their understanding of arrays and area/squared paper.





Vocabulary

Factor

Year 5

Developing understanding

Children should continue to count regularly, on and back, including steps of powers of 10. They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using x 10, x 20 etc.
- Doubling to solve x 2, x 4, x 8
- Recall of times tables
- Use of commutativity of multiplication

They continue to use the number line as an important image and informal jottings to their support thinking.

If children know the times table facts to 12 x 12, can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)?

Do they fully understand the effect of multiplying by 10, 100, 1000, including decimals?

Can they use practical resources and jottings to explore equivalent statements, e.g. 4 x 35 = 2 x 2 x 35?

They can identify factor pairs for numbers, recall prime numbers up to 19 and identify prime numbers up to 100 (with reasoning).

They should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

□ 2 x 50 = 1600   32 x □ □ = 1600   □ □ x 8 □ = 1600   □ □ □ x 4 = 1600

Children solve practical problems where they need to scale up by relating to known facts.

Written methods (progressing to 4d x 2d)

Children to explore how the grid method supports an understanding of short multiplication initially before progressing to long multiplication (for 2d x 2d) as shown below.

x	100	20	3
7	700	140	21

		1	2	3	
	x			7	
			2	1	
		1	4	0	
		7	0	0	
		8	6	1	

Progressing to long multiplication and decimals

	10	8
10	100	80
3	30	24

		1	8	
	x	1	3	
		5 $_2$	4	
	1	8	0	
	2	3	4	

Vocabulary

Cube numbers, prime numbers, square numbers, common factors, prime factors, composite numbers

Year 6

Developing understanding

Children should experiment with the order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45.

They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using x 10, x 20 etc.
- Doubling to solve x 2, x 4, x 8
- Recall of times tables
- Use of commutativity of multiplication

If children know the times table facts to 12 x 12, can they use this to recite other times tables, e.g. the 13 times tables or the 24 times table?

They should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

12.3 x □ = 86.1   12. □ x 3 = 86.1   12.3 x 3 = 8□. □

The children should identify common factors and multiples of given numbers.

They should solve practical problems where they need to scale-up by relating to known facts.

Written methods

Continue to refine and deepen understanding of written methods including fluency for using short and long multiplication with increasingly larger numbers and decimals.

	2	9	.	8					8	5	3	7	
x				6			x				2	9	
1	7	8	.	8				7	6_4	8_3	3_6	3	
1	5	4					1	7_1	0	7_1	4	0	
							2	4	7	5	7	3	
							1		1				

Vocabulary

Consolidate previous years

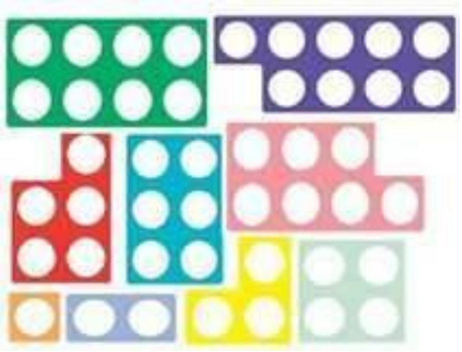
Progression from Mental Methods to Written Methods for Division

To carry out written methods of division successful, children need to be able to:

- understand division as repeated subtraction
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92
- multiply a two-digit number by a single-digit number mentally
- subtract numbers using the column method.

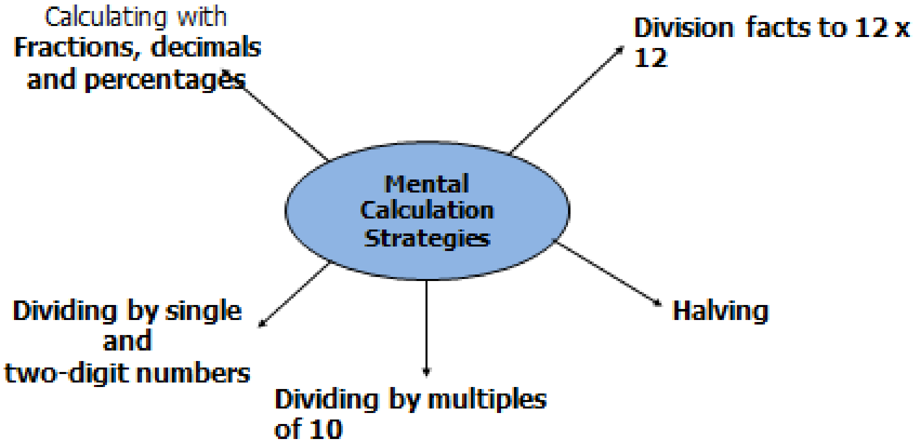
Mental Skills

Recognise the size and position of numbers
Count back in different steps 2s, 5s, 10s
Halve numbers to 20
Recognise division as repeated subtraction
Quick recall of division facts
Use known facts to derive associated facts
Divide by 10, 100, 1000 and understanding the effect
Divide by multiples of 10



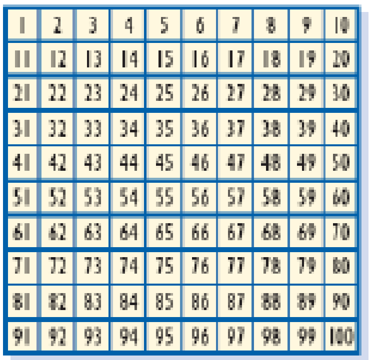
It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

Range of mental calculation strategies

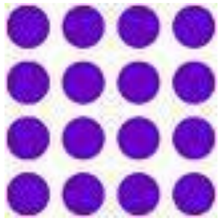


Concrete resources and visual representations

Counting apparatus
Place value apparatus
Arrays
100 squares
Number tracks
Numbered number lines
Marked but unnumbered lines
Empty number lines
Multiplication squares
Bead strings
Numicon including software
Resources from Times Tables Rockstars and NCETM 'Small –step teaching guidance'



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



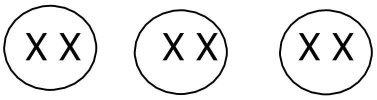
EYFS

The ELG states that children solve problems, including halving and sharing. Children will therefore need to see and hear representations of division as finding a fraction and sharing.

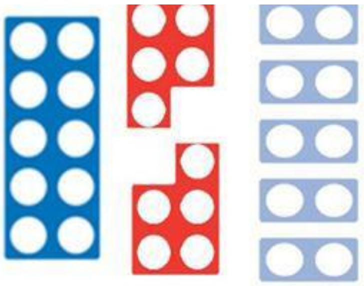
They will experience the language of sharing early on and develop an understanding of fairness; sharing of toys, fruit etc., and will have experienced the idea of groups – by working in a group with an adult or sorting toys or objects into groups of the same colour for instance.

Children begin with mostly pictorial representations linked to real life contexts:

Mum has 6 socks. She puts them into pairs – how many pairs did she make?

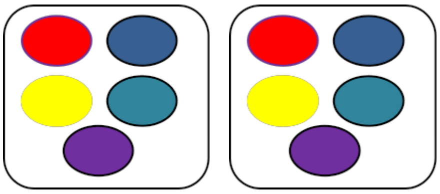


Numicon can be used to find how many smaller Numicon Shapes are the same as a larger Shape, e.g. 5 2s will fit over a 10 Shape.



Problem solving

I have 10 sweets. I want to share them with my friend. How many will we have each? (subtraction of 1)



Children have a go at recording the calculation that has been carried out.

Vocabulary

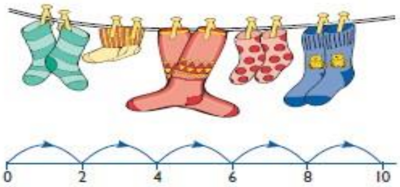
halve, share, share equally, left, left over

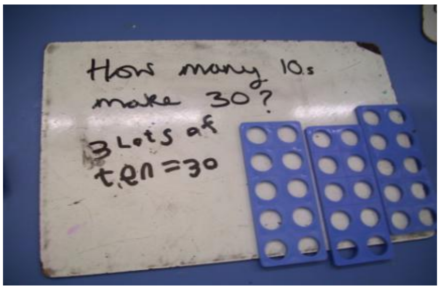
Year 1

Developing understanding

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



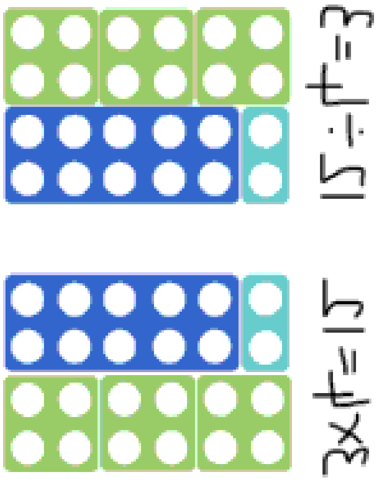


Children should begin to understand division as both sharing and grouping.

Sharing (generally subtraction of 1) – 6 sweets are shared between 2 people. How many do they have each?

Grouping (generally removal of a group) – How many 2's are in 6?

Problem solving



They use objects to group and share amounts to deve division in a practical sense, e.g. using Numicon Shapes 4s there are in 12?

How many pairs of gloves if you have 12 gloves?

Children should begin to explore finding ½ and ¼ and simple fractions of objects, numbers and quantities, e.g. 16 children went to the park at the weekend. Half of them went swimming. How many children went swimming?

Children should be given opportunities to reason about what they notice in number patterns.

Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, group in pairs, tens, equal groups of, divide, divide by, divide into

Year 2

Developing understanding

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.

Those who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 3 × 5 (five threes), hold up 5 fingers:

Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

They should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

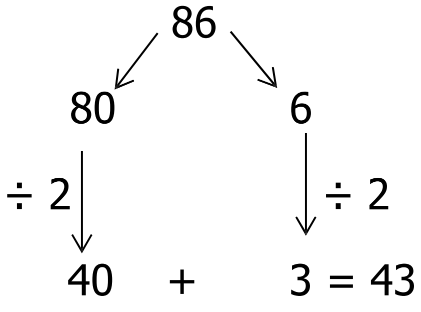
6 ÷ 2 = □   □ = 6 ÷ 2   □ ÷ 2 = 3   3 = □ ÷ ∇

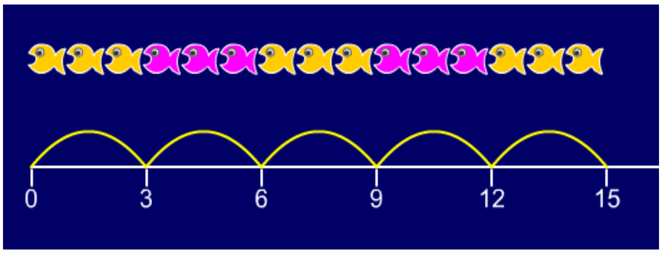
They should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

Towards written methods

Use number lines and jottings to develop an understanding of halving two digit numbers and solving division as counting equal groups (additive grouping is generally easier than subtractive grouping).





Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects is related to sharing.

They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half. Use the children's intuition to support their understanding of fractions as an answer to a sharing problem.

Vocabulary

Group in pairs, 3s … 10s etc., equal groups of, divide, ÷, divided by, divided into, remainder

Year 3

Developing understanding

Children should count regularly, on and back, in steps of 3, 4 and 8.

They are encouraged to use what they know about known times table facts to work out other times tables. This then helps them to make new connections (e.g. through halving they make connections between the 2, 4 and 8 times tables).

Children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

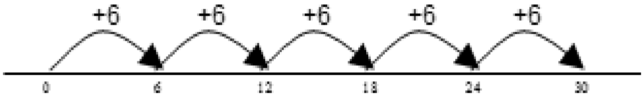
30 x □ = 60   □□ ÷ 3 = 20   □ = 60 ÷ 30   60 = □ ÷ ∇

They should be given opportunities to solve grouping and sharing problems practically, including where there is a remainder but the answer needs to be given as a whole number (adjusted) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

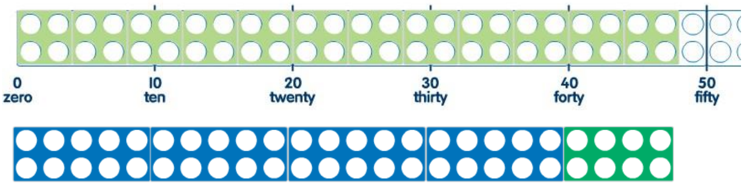
Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

Towards written methods

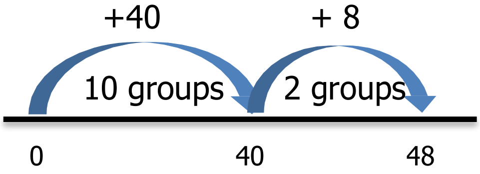
How many 6's are in 30 can be modelled as:

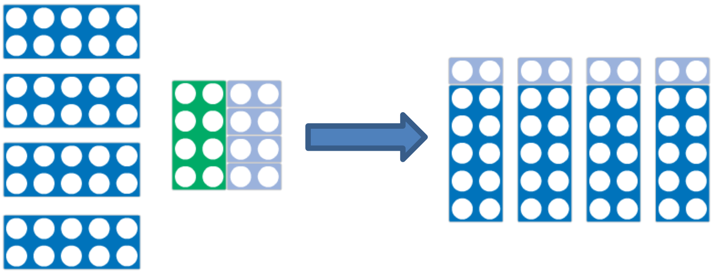


Children need to be able to partition the dividend in different ways.

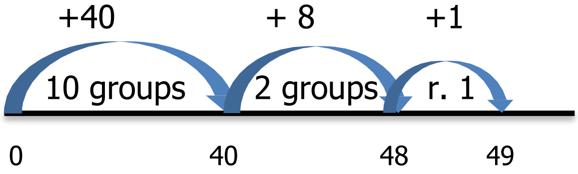


48 ÷ 4 = 12





49 ÷ 4 = 12 r1



Place value counters can be used to support children apply their knowledge of grouping and sharing (like Numicon Shapes the counters are a grouped resource):

600 ÷ 100 = How many groups of 100 in 600?

Vocabulary

Inverse, consolidate language from Y1 and Y2

Year 4

Developing understanding

Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000.

Children should learn the multiplication facts to 12 x 12 and corresponding division facts

Children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

30 x □□ = 600   □□□ ÷ 3 = 200   □□ = 600 ÷ 30   600 = □□ ÷ ∇∇

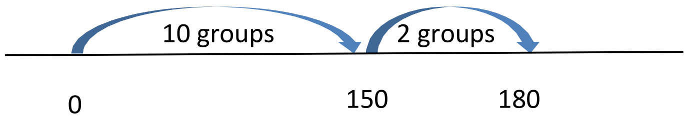
Towards written methods

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. They should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
  1. Dividend just over 10 x the divisor, e.g. 84 ÷ 7
  2. Dividend just over 10 x the divisor when the divisor is a teen number, e.g. 180 ÷ 15

All of the above stages should include calculations with remainders as well as without. Remainders should be interpreted according to the context, i.e. rounded up or down to relate to the answer to the problem.

e.g. 180 ÷ 15 = 12



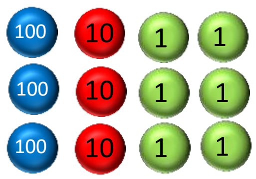
Jottings
15 x 2 = 30
15 x 5 = 75

15 x 10 = 150

Formal written methods

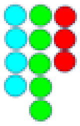
Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number. As with all standard methods, the numbers chosen should reflect the progression, e.g. use numbers that do not require an exchange or 'carry' until the understanding is secure.

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends.

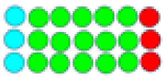


$$\begin{array}{r|l} & 1\ 1\ 2 \\ \hline 3 & 3\ 3\ 6 \end{array}$$

453 ÷ 3 =



exchange 1 blue into 10 green



$$\begin{array}{r|l} & 151 \\ \hline 3 & 4^{1}53 \end{array}$$

Vocabulary

Divide, divided by, divisible by, divided into, share between, groups of, factor, factor pair, multiple, times, as (big, long, wide …etc.), equals, remainder, quotient, divisor, inverse

Year 5

Developing understanding

Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency.
Children should learn and apply the multiplication facts to 12 x 12 and corresponding division facts.
Children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

30 x □□□ = 635 □□□ ÷ 9 = 200 □□□ = 635 ÷ 30 6324 = □□□ ÷ ∇∇

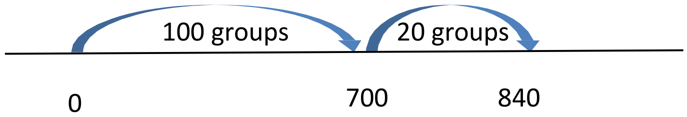
Towards written methods

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. They should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
  1. Dividend just over 10 x the divisor, e.g. 84 ÷ 7
  2. Dividend just over 10 x the divisor when the divisor is a teen number, e.g. 173 ÷ 15
  3. Dividend over 100 x the divisor, e.g. 840 ÷ 7
  4. Dividend over 20 x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without. Remainders should be interpreted according to the context, i.e. rounded up or down to relate to the answer to the problem.

e.g. 840 ÷ 7 = 120



Jottings

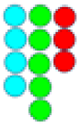
7 x 100 = 700
7 x 10 = 70

7 x 20 = 140

Formal Written Methods

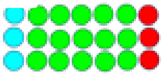
Continued to be developed with an understanding of place value and the language of sharing and grouping.

453 ÷ 3 =

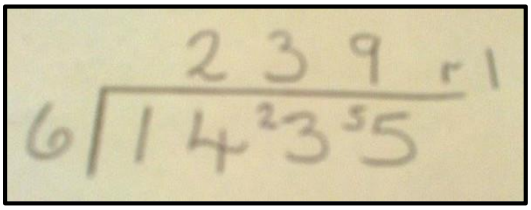


exchange 1 blue into 10 green

1435 ÷ 6



$$\begin{array}{r|l} & 151 \\ \hline 3 & 4^{1}53 \end{array}$$



239 r1
6|1 4²3⁵5

Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Vocabulary

Common factors, prime number, prime factors, composite numbers, short division, square number, cube number, inverse, power of

Year 6

Developing understanding

Children should continue to count regularly rehearsing all the steps covered in Y3 – Y5. Children should apply the multiplication facts to 12 x 12 and corresponding division facts. Children should use their understanding of the inverse and practical resources to solve missing number problems. They should use their knowledge of the rules of divisibility.

Towards written methods

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line when appropriate.

Quotients should be expressed as decimals and fractions.

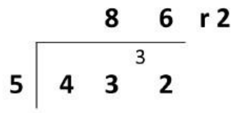
Formal Written Methods

Continued to be developed with an understanding of place value and the language of sharing and grouping. **If necessary, children will still support their thinking with place value counters until they are secure with the methods.**

They will express the remainder as a decimal or a fraction.

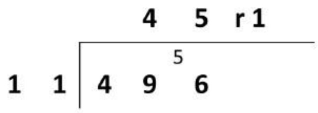
Short division is used when the divisor is a known 'table' i.e. 2 - 12

432 ÷ 5 becomes



Answer: 86 remainder 2

496 ÷ 11 becomes

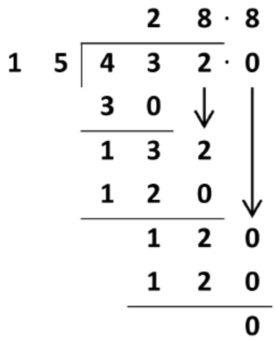


Answer: $45\frac{1}{11}$

Long division The 'arrow' method builds on a secure understanding of short division. It ignores the value of the digit. It is the preferred method for the end of Year 6. However if children have difficulty progressing to this method you may choose to use the long division method that links to chunking.

432 ÷ 15 becomes

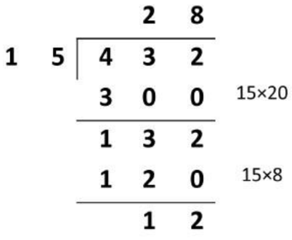
Arrow method



Answer: 28·8

432 ÷ 15 becomes

'chunking'



$\frac{12}{15} = \frac{4}{5}$

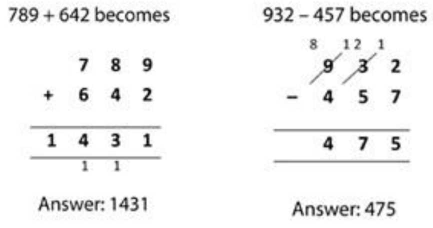
Answer: $28\frac{4}{5}$

Vocabulary

Consolidate previous years

Glossary of Terms

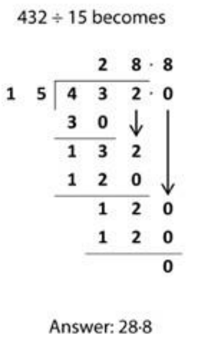
(taken from Mathematics glossary for teachers in Key Stages 1 to 3 published by NCETM January 2014)

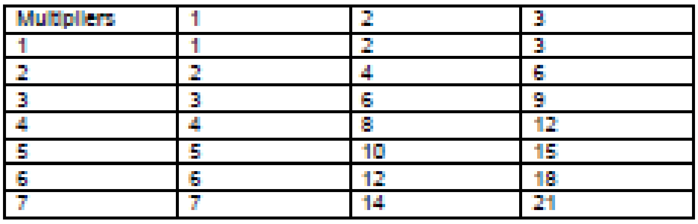
addend (KS1)	A number to be added to another. See also **dividend**, **subtrahend** and **multiplicand**.
addition (KS1)	The binary operation of addition on the set of all real numbers that adds one number of the set to another in the set to form a third number which is also in the set. The result of the addition is called the **sum** or **total**. The operation is denoted by the + sign. When we write 5 + 3 we mean 'add 3 to 5'; we can also read this as '5 plus 3'. In practice the order of addition does not matter: The answer to 5 + 3 is the same as 3 + 5 and in both cases the sum is 8. This holds for all pairs of numbers and therefore the operation of addition is said to be **commutative**. To add three numbers together, first two of the numbers must be added and then the third is added to this intermediate sum. For example, (5 + 3) + 4 means 'add 3 to 5 and then add 4 to the result' to give an overall total of 12. Note that 5 + (3 + 4) means 'add the result of adding 4 to 3 to 5' and that the total is again 12. The brackets indicate a priority of sub-calculation, and it is always true that (a + b) + c gives the same result as a + (b + c) for any three numbers a, b and c. This is the **associative** property of addition. Addition is the **inverse** operation to subtraction, and vice versa. There are two models for addition: **Augmentation** is when one quantity or measure is increased by another quantity. i.e. "I had £3.50 and I was given £1, then I had £4.50". **Aggregation** is the combining of two quantities or measures to find the total. E.g. "I had £3.50 and my friend had £1, we had £4.50 altogether.
array (KS1)	An ordered collection of counters, numbers etc. in rows and columns.
associative (KS1)	A binary operation ∗ on a set S is associative if a ∗ (b ∗ c) = (a ∗ b) ∗ c for all a, b and c in the set S. Addition of real numbers is associative which means a + (b + c) = (a + b) + c for all real numbers a, b, c. It follows that, for example, 1 + (2 + 3) = (1 + 2) + 3. Similarly multiplication is associative. Subtraction and division are not associative because: 1 – (2 – 3) = 1 – (−1) = 2, whereas (1 – 2) – 3 = (−1) – 3 = −4 and 1 ÷ (2 ÷ 3) = 1 ÷ 2/3 = 3/2, whereas (1 ÷ 2) ÷ 3 = (1/2) ÷ 3 = 1/6.
binary operation (KS1)	A rule for combining two numbers in the set to produce a third also in the set. Addition, subtraction, multiplication and division of real numbers are all binary operations.
brackets (KS2)	Symbols used to group numbers in arithmetic or letters and numbers in algebra and indicating certain operations as having priority. Example: 2 x (3 + 4) = 2 x 7 = 14 whereas 2 x 3 + 4 = 6 + 4 = 10. Example: 3(x + 4) denotes the result of adding 4 to a number and then multiplying by 3; (x + 1)2 denotes the result of adding 1 to a number and then squaring the result
columnar addition or subtraction (KS2)	A **formal method** of setting out an addition or a subtraction in ordered columns with each column representing a decimal place value and ordered from right to left in increasing powers of 10. With addition, more than two numbers can be added together using column addition, but this extension does not work for subtraction.  (Examples taken from Appendix 1 of the Primary National Curriculum for Mathematics)
common factor (KS2)	A number which is a factor of two or more other numbers, for example 3 is a common factor of the numbers 9 and 30 This can be generalised for algebraic expressions: for example (x – 1) is a common factor of (x – 1)2 and (x – 1)(x + 3).
common multiple (KS2)	An integer which is a multiple of a given set of integers, e.g. 24 is a common multiple of 2, 3, 4, 6, 8 and 12.

commutative (KS1)	A binary operation ∗ on a set S is commutative if a ∗ b = b ∗ a for all a and b ∈ S. Addition and multiplication of real numbers are commutative where a + b = b + a and a × b = b × a for all real numbers a and b. It follows that, for example, 2 + 3 = 3 + 2 and 2 x 3 = 3 x 2. Subtraction and division are not commutative since, as counter examples, 2 – 3 ≠ 3 – 2 and 2 ÷ 3 ≠ 3 ÷ 2.
compensation (in calculation) (KS1/2)	A mental or written calculation strategy where one number is rounded to make the calculation easier. The calculation is then adjusted by an appropriate compensatory addition or subtraction. Examples: • 56 + 38 is treated as 56 + 40 and then 2 is subtracted to compensate. • 27 × 19 is treated as 27 × 20 and then 27 (i.e. 27 × 1) is subtracted to compensate. • 67 − 39 is treated as 67 − 40 and then 1 is added to compensate.
complement (in addition) (KS2)	In addition, a number and its complement have a given total. Example: When considering complements in 100, 67 has the complement 33, since 67 + 33 = 100
concrete objects (KS1)	Objects that can be handled and manipulated to support understanding of the structure of a mathematical concept. Materials such as Dienes (Base 10 materials), Cuisenaire, Numicon, pattern blocks are all examples of concrete objects.
correspondence problems (KS2)	Correspondence problems are those in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
cube (KS1/2)	In number and algebra, the result of multiplying to power of three, n3 is read as 'n cubed' or 'n to the power of three' Example: Written 23, the cube of 2 is (2 x 2 x 2) = 8.
cube number (KS2)	A number that can be expressed as the product of three equal integers. Example: 27 = 3 × 3 × 3. Consequently, 27 is a cube number; It is the cube of 3 or 3 cubed. This is written compactly as 27 = 3^3, using index, or power, notation.
cube root (KS3)	A value or quantity whose cube is equal to a given quantity. Example: the cube root of 8 is 2 since 23 = 8. This is recorded as 3√ 8 = 2 or 8 1/3 = 2
decomposition (KS2)	See **subtraction** by decomposition
degree of accuracy (KS2)	A measure of the precision of a calculation, or the representation of a quantity. A number may be recorded as accurate to a given number of decimal places, or rounded to the nearest integer, or to so many significant figures.
difference (KS1)	In mathematics (as distinct from its everyday meaning), difference means the numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another. E.g. the difference between 12 and 5 is 7; 12 is 5 more than 7 or 7 is 5 fewer than 12. Difference is one way of thinking about subtraction and can, in some circumstances, be a more helpful image for subtraction than 'take-away' e.g. 102 - 98
distributive (KS2)	One binary operation ∗ on a set S is distributive over another binary operation • on that set if a ∗ (b • c) = (a ∗ b) • (a ∗ c) for all a, b and c ∈ S. For the set of real numbers, multiplication is distributive over addition and subtraction since a(b + c) = ab + ac for all a, b and c real numbers. It follows that 4(50 + 6) = (4 x 50) + (4 x 6) and 4 x (50 – 2) = (4 x 50) – (4 x 2). For division $\frac{(a + b)}{c} = \frac{a}{c} + \frac{b}{c}$ (division is distributive over addition) But $\frac{c}{(a+b)} \neq \frac{c}{a} + \frac{c}{b}$ (addition is not distributive over division) Addition, subtraction and division are not distributive over other number operations.
divide (KS1)	To carry out the operation of division.
dividend (KS1)	In division, the number that is divided. E.g. in 15 ÷ 3, 15 is the dividend See also Addend, subtrahend and multiplicand.

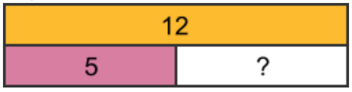
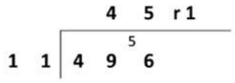
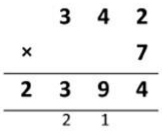
divisibility (KS2)	The property of being divisible by a given number. Example: A test of divisibility by 9 checks if a number can be divided by 9 with no remainder.
divisible (by) (KS2)	A whole number is divisible by another if there is no remainder after division and the result is a whole number. Example: 63 is divisible by 7 because 63 ÷ 7 = 9 remainder 0. However, 63 is not divisible by 8 because 63 ÷ 8 = 7.875 or 7 remainder 7.
division (KS1)	1. An operation on numbers interpreted in a number of ways. Division can be sharing – the number to be divided is shared equally into the stated number of parts; or grouping – the number of groups of a given size is found. Division is the inverse operation to multiplication. 2. On a scale, one part. Example: Each division on a ruler might represent a millimetre.
divisor (KS2)	The number by which another is divided. Example: In the calculation 30 ÷ 6 = 5, the divisor is 6. In this example, 30 is the dividend and 5 is the quotient.
double (KS1)	1. To multiply by 2. Example: Double 13 is (13 × 2) = 26. 2. The number or quantity that is twice another. Example: 26 is double 13. In this context, a 'near double' is one away from a double. Example: 27 is a near double of 13 and of 14. (N.B. spotting near doubles can be a useful mental calculation strategy e.g. seeing 25 + 27 as 2 more than double 25.
efficient methods (KS2)	A means of calculation (which can be **mental or written**) that achieves a correct answer with as few steps as possible. In written calculations this often involves setting out calculations in a columnar layout. If a calculator is used the most efficient method uses as few key entries as possible.
equal (KS1)	Symbol: =, read as 'is equal to' or 'equals'. and meaning 'having the same value as'. Example: 7 – 2 = 4 + 1 since both expressions, 7 – 2 and 4 + 1 have the same value, 5.
equation (KS3)	A mathematical statement showing that two expressions are equal. The expressions are linked with the symbol =. Examples: 7 – 2 = 4 + 1 4x = 3 x^2 – 2x + 1 = 0
error (KS3)	1. The difference between an accurate calculation and an approximate calculation or estimate; the difference between an exact representation of a number and an approximation to it obtained by rounding or some other process. In a calculation, if all numbers are rounded to some degree of accuracy the errors become more significant. 2. A mistake
estimate (KS2)	1. Verb: To arrive at a rough or approximate answer by calculating with suitable approximations for terms or, in measurement, by using previous experience. 2. Noun: A rough or approximate answer.
even number (KS1)	An integer that is divisible by 2.
exchange (KS2)	Change a number or expression for another of equal value. The process of exchange is used in some standard compact methods of calculation. Examples: 'carrying figures' in addition, multiplication or division; and '**decomposition**' in subtraction.
exponent (KS3)	Also known as index, a number, positioned above and to the right of another (the base), indicating repeated multiplication when the exponent is a positive integer. Example 1: n^2 indicates n × n; and 'n to the (power) 4', that is n^4 means n × n × n × n. Example 2: since 2^5 = 32 we can also think of this as '32 is the fifth power of 2'. Any positive number to power 1 is the number itself; x^1 = x, for any positive value of x. Exponents may be negative, zero, or fractional. Negative integer exponents are the reciprocal of the corresponding positive integer exponent, for example, 2^{-1} = ½. Any positive number to power zero equals 1; x^0 = 1, for any positive value of x. The positive unit fractional powers represent roots, which are the inverse to the corresponding integer powers; thus 81/3 = $\sqrt[3]{8}$ = 2, since 2^3 = 8 Note: Power notation is not used for zero, since division by zero is undefined.
expression (KS2)	A mathematical form expressed symbolically. Examples: 7 + 3; a2 + b2.
factor (KS2)	When a number, or polynomial in algebra, can be expressed as the product of two numbers or polynomials, these are factors of the first. Examples: 1, 2, 3, 4, 6 and 12 are all factors of 12 because 12 = 1 × 12 = 2 × 6 = 3 × 4: (x – 1) and (x + 4) are factors of (x2 + 3x − 4) because (x – 1)(x + 4) = (x2 + 3x – 4)
factorise (KS2)	To express a number or a polynomial as the product of its factors. Examples: Factorising 12: 12 = 1 × 12 = 2 × 6 = 3 × 4

	The factors of 12 are 1, 2, 3, 4, 6 and 12. 12 may be expressed as a product of its prime factors: 12 = 2 × 2 × 3 Factorising x2 – 4x – 21: x2 – 4x – 21 = (x + 3) (x – 7) The factors of x2 – 4x – 21 are (x + 3) and (x – 7)
facts (KS1)	i.e. Multiplication / division/ addition/ subtraction facts. The word 'fact' is related to the four operations and the instant recall of knowledge about the composition of a number. i.e. an addition fact for 20 could be 10+10; a subtraction fact for 20 could be 20-9=11. A multiplication fact for 20 could be 4 x 5 and a division fact for 20 could be 20÷5 = 4.
fluency (KS1)	To be mathematically fluent one must have a mix of conceptual understanding, procedural fluency and knowledge of facts to enable you to tackle problems appropriate to your stage of development confidently, accurately and efficiently.
formal written methods (KS2)	Setting out working in columnar form. In multiplication, the formal methods are called short or long multiplication depending on the size of the numbers involved. Similarly, in division the formal processes are called short or long division. See Mathematics Appendix 1 in the 2013 National Curriculum.
(the) **four operations**	Common shorthand for the four arithmetic operations of addition, subtraction, multiplication and division.
general statement (KS1)	A statement that applies correctly to all relevant cases. e.g. the sum of two odd numbers is an even number.
generalise (KS1)	To formulate a general statement or rule.
highest common factor (HCF) (KS3)	The common factor of two or more numbers which has the highest value. Example: 16 has factors 1, 2, 4, 8, 16. 24 has factors 1, 2, 3, 4, 6, 8, 12, 24. 56 has factors 1, 2, 4, 7, 8, 14, 28, 56. The common factors of 16, 24 and 56 are 1, 2, 4 and 8. Their highest common factor is 8.
inequality (KS1)	When one number, or quantity, is not equal to another. Statements such as a ≠ b, a<b, a ≤, b, a > b or a≥b are inequalities. The inequality signs in use are: ≠ means 'not equal to'; A ≠ B means 'A is not equal to B" < means 'less than'; A < B means 'A is less than B' > means 'greater than'; A > B means 'A is greater than B' ≤ means 'less than or equal to'; A ≤ B means 'A is less than or equal to B' ≥ means 'greater than or equal to'; A ≥ B means 'A is greater than or equal to B'
integer (KS2)	Any of the positive or negative whole numbers and zero. Example: …-2, -1, 0, +1, +2 … The integers form an infinite set; there is no greatest or least integer.
inverse operations (KS1)	Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. 5 + 6 – 6 = 5. Multiplication and division are inverse operations e.g. 6 × 10 ÷ 10 = 6. Squaring and taking the square root are inverse to each other: √x2 = (√x)2 = x; similarly with cube and cube root, and any integer power n and nth root. Some operations, such as reflection in the x-axis, or 'subtract from 10' are self-inverse i.e. they are inverses of themselves
irrational number (KS3)	A number that is not an integer and cannot be expressed as a common fraction with a non-zero denominator. Examples: √ 3 and π. Real irrational numbers, when expressed as decimals, are infinite, non-recurring decimals.
least common multiple (LCM) (KS3)	The common multiple of two or more numbers, which has the least value. Example: 3 has multiples 3, 6, 9, 12, 15, 18, 21, 24 …, 4 has multiples 4, 8, 12, 16, 20, 24 … and 6 has multiples 6, 12, 18, 24, 30 …. The common multiples of 3, 4 and 6 include 12, 24 and 36. The least common multiple of 3, 4 and 6 is 12.
long division (KS2)	A columnar algorithm for division by more than a single digit, most easily described with

<table>
<tr><td></td><td>
an example:</td></tr>
<tr><td>long multiplication
(KS2)</td><td>A columnar algorithm for pe ication by more than a single digit, again best illustrated by an example:

(Example taken from Appendix 1 of the Primary National Curriculum for Mathematics)</td></tr>
<tr><td>mental calculation
(KS1)</td><td>Referring to calculations that are largely carried out mentally, but may be supported with a few simple written jottings.</td></tr>
<tr><td>minus
(KS1)</td><td>A name for the symbol −, representing the operation of subtraction.</td></tr>
<tr><td>missing number problems
(KS1)</td><td>A problem of the type 7 = □ − 9 often used as an introduction to algebra.</td></tr>
<tr><td>multiple
(KS1)</td><td>For any integers a and b, a is a multiple of b if a third integer c exists so that a = bc
Example: 14, 49 and 70 are all multiples of 7 because 14 = 7 x 2, 49 = 7 x 7 and 70 = 7 x 10.. -21 is also a multiple of 7 since -21 = 7 ×-3.</td></tr>
<tr><td>multiplicand
(KS1)</td><td>A number to be multiplied by another.
e.g. in 5 × 3, 5 is the multiplicand as it is the number to be multiplied by 3.
See also Addend, subtrahend and dividend.</td></tr>
<tr><td>multiplication
(KS1)</td><td>Multiplication (often denoted by the symbol "×") is the mathematical operation of scaling one number by another. It is one of the four binary operations in arithmetic (the others being addition, subtraction and division).
Because the result of scaling by whole numbers can be thought of as consisting of some number of copies of the original, whole-number products greater than 1 can be computed by repeated addition; for example, 3 multiplied by 4 (often said as "3 times 4") can be calculated by adding 4 copies of 3 together:
3 x 4 = 3 + 3 + 3 + 3 = 12
Here 3 and 4 are the "factors" and 12 is the "product".
Multiplication is the inverse operation of division, and it follows that
7 ÷ 5 × 5 = 7
Multiplication is commutative, associative and distributive over addition or subtraction.</td></tr>
<tr><td>multiplication table
(KS1)</td><td>An array setting out sets of numbers that multiply together to form the entries in the array, for example

<table>
<tr><th>Multipliers</th><th>1</th><th>2</th><th>3</th></tr>
<tr><td>1</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>2</td><td>2</td><td>4</td><td>6</td></tr>
<tr><td>3</td><td>3</td><td>6</td><td>9</td></tr>
<tr><td>4</td><td>4</td><td>8</td><td>12</td></tr>
<tr><td>5</td><td>5</td><td>10</td><td>15</td></tr>
<tr><td>6</td><td>6</td><td>12</td><td>18</td></tr>
<tr><td>7</td><td>7</td><td>14</td><td>21</td></tr>
</table></td></tr>
<tr><td>multiplicative reasoning
(KS2)</td><td>Multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts.</td></tr>
</table>

	For example, from this: 3 bags of sweets, 8 sweets in each bag. How many sweets? To this and beyond: Julie bought a dress in a sale for £49.95 after it was reduced by 30%. How much would she have paid before the sale?
multiply (KS1)	Carry out the process of multiplication.
notation (KS1)	A convention for recording mathematical ideas. Examples: Money is recorded using decimal notation e.g. £2.50 Other examples of mathematical notation include $a + a = 2a$, $y = f(x)$ and $n \times n \times n = n^3$,
number bond (KS1)	A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10.
number line (KS1)	A line where numbers are represented by points upon it.
number sentence (KS1)	A mathematical sentence involving numbers. Examples: $3 + 6 = 9$ and $9 > 3$
number track (KS1)	A numbered track along which counters might be moved. The number in a region represents the number of single moves from the start.
odd number (KS2)	An integer that has a remainder of 1 when divided by 2.
order of magnitude (KS2)	The approximate size, often given as a power of 10. Example of an order of magnitude calculation: $95 \times 1603 \div 49 \approx 102 \times 16 \times 102 \div(5 \times 101) \approx 3 \times 103$
order of operation (KS2)	This refers to the order in which different mathematical operations are applied in a calculation. Without an agreed order an expression such as $2 + 3 \times 4$ could have two possible values: $5 \times 4 = 20$ (if the operation of addition is applied first) $2 + 12 = 14$ (if the operation of multiplication is applied first) The agreed order of operations is that: • Powers or indices take precedent over multiplication or division – $2 \times 32 = 18$ not 25; • Multiplication or division takes precedent over addition and subtraction – $2 + 3 \times 4 = 14$ not 20 • If brackets are present, the operation contained therein always takes precedent over all others – $(2 + 3) \times 4 = 20$ This convention is often encapsulated in the mnemonic BODMAS or BIDMAS: Brackets Orders / Indices (powers) Division & Multiplication Addition & Subtraction
partition (KS1)	1. To separate a set into subsets. 2. To split a number into component parts. Example: the two-digit number 38 can be partitioned into 30 + 8 or 19 + 19. 3. A model of division. Example: 21 ÷ 7 is treated as 'how many sevens in 21?'
pictorial representations (KS1)	Pictorial representations enable learners to use pictures and images to represent the structure of a mathematical concept. The pictorial representation may build on the familiarity with concrete objects. E.g. a square to represent a Dienes 'flat' (representation of the number 100). Pupils may interpret pictorial representations provided to them or create a pictorial representation themselves to help solve a mathematical problem.
plus (KS1)	A name for the symbol +, representing the operation of addition.
power (of ten) (KS2)	1. 100 (i.e. 10^2 or 10 x 10) is the second power of 10, 1000 (i.e.10^3 or 10 x 10 x 10) is the third power of 10 etc. Powers of other numbers are defined in the same way. Example: 2 (2^1), 4 (2^2), 8 (2^3), 16 (2^4) etc are powers of 2. 2. A fractional power represents a root. Example: x½ = √x 3. A negative power represents the reciprocal. Example: x−1 = 1/x 4. By convention any number or variable to the power 0 equals 1. i.e. $x^0 = 1$
prime factor (KS2)	The factors of a number that are prime. Example: 2 and 3 are the prime factors of 12 (12 = 2 x 2 x 3). See also factor.

prime factor decomposition (KS2)	The process of expressing a number as the product of factors that are prime numbers. Example: 24 = 2 × 2 × 2 × 3 or 2^3 × 3. Every positive integer has a unique set of prime factors.
prime number (KS2)	A whole number greater than 1 that has exactly two factors, itself and 1. Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1).
priority of operations (KS2)	Generally, multiplication and division are done before addition and subtraction, but this can be ambiguous, so brackets are used to indicate calculations that must be done before the remainder of the operations are carried out. See order of operation
product (KS1)	The result of multiplying one number by another. Example: The product of 2 and 3 is 6 since 2 × 3 = 6.
quotient (KS2)	The result of a division. Example: 46 ÷ 3 = 15⅓ and 15⅓ is the quotient of 46 by 3. Where the operation of division is applied to the set of integers, and the result expressed in integers, for example 46 ÷ 3 = 15 remainder 1 then 15 is the quotient of 46 by 3 and 1 is the remainder.
recurring decimal (KS2)	A decimal fraction with an infinitely repeating digit or group of digits. Example: The fraction ⅓ is the decimal 0.33333 …, referred to as nought point three recurring and may be written as 0.3 (with a dot over the three). Where a block of numbers is repeated indefinitely, a dot is written over the first and last digit in the block e.g. 1/7 = 0.˙142857˙
remainder (KS2)	In the context of division requiring a whole number answer (quotient), the amount remaining after the operation. Example: 29 divided by 7 = 4 remainder 1.
repeated addition (KS1)	The process of repeatedly adding the same number or amount. One model for multiplication. Example 5 + 5 + 5 + 5 = 5 x 4.
repeated subtraction(KS1)	The process of repeatedly subtracting the same number or amount. One model for division. Example 35 -5 - 5 - 5 - 5 - 5 - 5 - 5 = 0 so 35 ÷ 5 = 7 remainder 0.
representation (KS2)	The word 'representation' is used in the curriculum to refer to a particular form in which the mathematics is presented, so for example a quadratic function could be expressed algebraically or presented as a graph; a quadratic expression could be shown as two linear factors multiplied together or the multiplication could be expanded out; a probability distribution could be presented in a table or represented as a histogram, and so on. Very often, the use of an alternative representation can shed new light on a problem. An array is a useful representation for multiplication and division which helps to see the inverse relationship between the two. The Bar Model is a useful representation of for many numerical problems. E.g. Tom has 12 sweet and Dini has 5. How many more sweets does Tom have than Dini? 
scale (verb) (KS2)	To enlarge or reduce a number, quantity or measurement by a given amount (called a scale factor). e.g. to have 3 times the number of people in a room than before; to find a quarter of a length of ribbon; to find 75% of a sum of money.
share (equally) (KS1)	Sections of this page that are currently empty will be filled over the coming weeks. One model for the process of division.
short division (KS2)	A compact written method of division. Example: 496 ÷ 11 becomes  Answer : 45 1/11 (Example taken from Appendix 1 of the Primary National Curriculum for Mathematics)
short multiplication (KS2)	Essentially, simple multiplication by a one digit number, with the working set out in columns. 342 x 7 becomes  Answer: 2394 (Example taken from Appendix 1 of the Primary National Curriculum for Mathematics)

sign (KS1)	A symbol used to denote an operation. Examples: addition sign +, subtraction sign −, multiplication sign ×, division sign ÷, equals sign = etc. In the case of directed numbers, the positive + or negative − sign indicates the direction in which the number is located from the origin along the number line.
square (KS1)	1. A quadrilateral with four equal sides and four right angles. 2. The square of a number is the product of the number and itself. Example: the square of 5 is 25. This is written $5^2 = 25$ and read as five squared is equal to twenty-five. See also square number and square root.
square number (KS2)	A number that can be expressed as the product of two equal numbers. Example 36 = 6 x 6 and so 36 is a square number or “6 squared”. A square number can be represented by dots in a square array.
square root (KS3)	A number whose square is equal to a given number. Example: one square root of 25 is 5 since $5^2 = 25$. The square root of 25 is recorded as $\sqrt{25} = 5$. However, as well as a positive square root, 25 has a negative square root, since $(-5)^2 = 25$.
subtract (KS1)	Carry out the process of subtraction
subtraction (KS1)	The inverse operation to addition. Finding the difference when comparing magnitude. Take away.
subtraction by decomposition (KS2)	A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. In this method the number to be subtracted from (the **minuend**) is re-partitioned, if necessary, in order that each digit of the number to be subtracted (the **subtrahend**) is smaller than its corresponding digit in the minuend. e.g. in 719 – 297, only the digits in the hundreds and the ones columns are bigger in the minuend than the subtrahend. By re-partitioning 719 into 6 hundreds, 11 tens and 9 ones each separate subtraction can be performed simply, i.e.: 9 – 7, 11 (tens) – 9 (tens) and 6 (hundreds) – 2 (hundreds). $\begin{array}{rrrr} & {}^{6}\cancel{7} & {}^{1}1 & 9 \\ - & 2 & 9 & 7 \\ \hline & 4 & 2 & 2 \end{array}$
subtraction by equal addition	A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. This method relies on the understanding that adding the same quantity to both the **minuend** and the **subtrahend** retains the same difference. This is a useful technique when a digit in the subtrahend is larger than its corresponding digit in the minuend. E.g. in the example below, 7 > 2, therefore 10 has been added to the 2 (in the ones place) of the minuend to make 12 (ones) and also added to the 5 (tens) of the subtrahend to make 60 (or 6 tens) before the first step of the calculation can be completed. Similarly 100 has been added to the 3 (tens) of the minuend to make 13 (tens) and also added to the 4 (hundreds) of the subtrahend to make 5 (hundred). 932-457 becomes $\begin{array}{rrrr} & 9 & {}^{1}3 & {}^{1}2 \\ - & \cancel{4}_{5} & \cancel{5}_{6} & 7 \\ \hline & 4 & 7 & 5 \\ \hline \end{array}$ Example taken from Appendix 1 of the Primary National Curriculum for Mathematics
subtrahend (KS1)	A number to be subtracted from another. See also **Addend**, dividend and multiplicand.
sum (KS1)	The result of one or more additions.
symbol (KS1)	A letter, numeral or other mark that represents a number, an operation or another mathematical idea. Example: L (Roman symbol for fifty), > (is greater than).
take away (KS1)	1. Subtraction as reduction 2. Remove a number of items from a set.
terminating decimal (KS2)	A decimal fraction that has a finite number of digits. Example: 0.125 is a terminating decimal. In contrast ⅓ is a recurring decimal fraction. All terminating decimals can be expressed as fractions in which the denominator is a multiple of 2 or 5.

total (KS1)	1. The aggregate. Example: the total population - all in the population. 2. The sum found by adding.
zero (KS1)	1. Nought or nothing; zero is the only number that is neither positive nor negative. 2. Zero is needed to complete the number system. In our system of numbers : a – a = 0 for any number a. a + (−a) = 0 for any number a; a + 0 = 0 + a = a for any number a; a – 0 = a for any number a; a × 0 = 0 × a = 0 for any number a; division by zero is not defined as it leads to inconsistency. 3. In a place value system, a place-holder. Example: 105. 4. The cardinal number of an empty set.